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Systematic Approach to the Analysis of NN and $N\bar{N}$ Total Cross Sections*

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A general, rigorous, and extremely simple method of analyzing nucleon-nucleon and nucleon-antinucleon total cross sections is presented. The method is valid for all energies and provides a simple link between the experimental quantities of fundamental physical interest. It is particularly appropriate in the high-energy region and, as an example, is applied to the Regge model. The results are derived by using the concept of crossing and a weak form of Mandelstam analyticity, and depend upon the observation that the NN and $N\bar{N}$ total cross sections can be expressed in terms of a single spin-triplet $N\bar{N}\rightarrow N\bar{N}$ transition amplitude. A basic ingredient is the experimental knowledge of $\sigma_{\bar{p}n}$ or, equivalently, $\sigma_{\bar{n}p}$. A primary aim of this work is to encourage the experimental measurement of these cross sections.

I. INTRODUCTION

THE use of the Mandelstam representation in strong-interaction physics has focused attention on the importance of the concept of crossing, i.e., of the interrelation between the various channels reached by analytic continuation from the region of a given process. In fact, it is probably true to say that (with the exception of those cases in which direct channel resonances dominate) almost all calculations in strong-interaction physics, and in particular in Regge-type models, either contain or evaluate as primary quantities the scattering amplitudes in the t channel.

At the same time it is well known that the study of nucleon-nucleon or nucleon-antinucleon scattering is greatly complicated by the spin-$\frac{1}{2}$ nature of the particles involved. Thus the scattering amplitude in each isotopic spin state is found to depend on five independent scalar functions of the energy and the angle of scattering.

The purpose of this paper is to point out a general, rigorous, and extremely simple method of analyzing NN and $N\bar{N}$ total cross sections. In this method we take advantage of the observation in the first paragraph and introduce a set of four experimental quantities, linear combinations of σ_{pp}, σ_{pn}, $\sigma_{\bar{p}p}$, and $\sigma_{\bar{p}n}$, which play a fundamental role because they are directly related to a single t-channel scattering amplitude.

The principal observation is that the total (unpolarized) NN or $N\bar{N}$ cross sections in the s channel can be expressed in terms of only *one* of the five $N\bar{N}\rightarrow N\bar{N}$ spin-transition amplitudes (f_1 through f_5 in the notation of Ref. 1) in the t channel. This transition can take place with parity $P=\pm 1$ and isospin $I=0, 1$. Each of the suitably chosen linear combinations of the four independent experimental cross sections then corresponds to a single t-channel transition with given P and I. A study of the energy dependence of these combinations should then prove extremely useful in the analysis and testing of any theoretical model of NN or $N\bar{N}$ scattering.

It is perhaps worth emphasizing that although we illustrate the method by applying it to the (t channel) Regge model, the basic results [Eqs. (24)] are completely general, are valid at all energies, and depend only on the optical theorem and a rather weak form of Mandelstam analyticity.

The analysis discussed here requires an experimental knowledge of the $\bar{p}n$ (or equivalently $\bar{n}p$) total cross section. It is hoped that the simplicity and usefulness of the method of analysis will stimulate a vigorous experimental attack on this important physical quantity.

In Sec. II, we go through the algebra leading to our results. The reader who is solely interested in using the proposed method of analysis can safely proceed to Eq. (24).

In Sec. III, we illustrate the method briefly by applying it to the Regge model.

II. DERIVATION OF RESULTS

Consider NN scattering in the s channel. We shall use the usual Mandelstam variables

$$s=4(m^2+p^2)\,,\quad t=-2p^2(1-\cos\theta)\,,\quad \text{and}\quad u=-2p^2(1+\cos\theta)\,,\tag{1}$$

where p is the momentum in the center of mass of the two nucleons. In terms of the laboratory system kinetic energy T, we have

$$s=4m^2+2mT\quad\text{and}\quad p=(mT/2)^{1/2}.\tag{2}$$

Following the notation of Ref. 1, we denote by $\phi_1, \cdots, \phi_5$ the five helicity amplitudes in the s channel. With the optical theorem we can relate the total unpolarized cross section to the imaginary parts of the forward, purely elastic amplitude. For a given isospin I, we have

$$\sigma_{NN}{}^I=(2\pi/p)\,\mathrm{Im}[\phi_1{}^I(t=0)+\phi_3{}^I(t=0)]\,,\tag{3}$$

as only ϕ_1 and ϕ_3 pertain to transitions in which neither nucleon flips its spin.

* Work done under the auspices of the U. S. Atomic Energy Commission.

[1] M. L. Goldberger, M. T. Grisaru, S. W. Mac Dowell, and D. Y. Wong, Phys. Rev. 120, 2250 (1960).

The next step is to use the well-known crossing matrices[1] to express the s channel ϕ_i in terms of singlet and triplet $N\bar{N}$ transition amplitudes f_1 through f_5 of the t channel. (Note that, in the notation of Ref. 1, these would be $\bar{f}_1, \cdots, \bar{f}_5$; we omit the bars for convenience.) The simplest way to do this is to introduce as auxiliary quantities the five scalar amplitudes $G_1, \cdots, G_5$. From Eqs. (4.23) of Ref. 1, we obtain

$$\phi_i{}^I = A_{ij} G_j{}^I , \tag{4}$$

where

$$A_{ij} = \frac{1}{4(s)^{1/2}} \begin{bmatrix} s & \dfrac{4m^2(u-t)}{t+u} & 4m^2 & \dfrac{4m^2(u-t)}{t+u} & t+u \\ -s & \dfrac{(u-t)(s-t-u)}{t+u} & -4m^2 & \dfrac{4m^2(u-t)}{t+u} & t+u \\ 0 & \dfrac{8m^2u}{t+u} & 2u & \dfrac{2su}{t+u} & 0 \\ 0 & \dfrac{8m^2t}{t+u} & -2t & \dfrac{2st}{t+u} & 0 \\ 0 & \dfrac{-4m(stu)^{1/2}}{t+u} & 0 & \dfrac{-4m(stu)^{1/2}}{t+u} & 0 \end{bmatrix} . \tag{5}$$

Also from Eqs. (2.9) and (4.24) of Ref. 1, we obtain

$$G_i{}^I(s,u,t) = (-1)^{i+I} G_i{}^I(s,t,u) . \tag{6}$$

(Note that we are using the notation u instead of $\bar{t}$ of Ref. 1.) And from Eq. (4.27) of Ref. 1,

$$G_i{}^I(s,u,t) = \Delta_{ij} B^{II'} G_j{}^{I'}(u,s,t) , \tag{7}$$

where

$$B = \tfrac{1}{2} \begin{pmatrix} -1 & 3 \\ 1 & 1 \end{pmatrix} \tag{8}$$

and

$$\Delta = \frac{1}{4} \begin{bmatrix} -1 & 6 & 4 & -4 & -1 \\ 1 & 2 & 0 & 0 & 1 \\ 1 & 0 & 2 & 2 & -1 \\ -1 & 0 & 2 & 2 & 1 \\ -1 & 6 & -4 & 4 & -1 \end{bmatrix} . \tag{9}$$

Interchanging t and u in Eq. (7) and then using Eq. (6), we obtain

$$G_i{}^I(s,u,t) = (-1)^{i+I} \Delta_{ij} B^{II'} G_j{}^{I'}(t,s,u) . \tag{10}$$

Equation (10) relates the G functions of the s channel to the G functions of the t channel. Combining Eqs. (10), (4), and (5), we obtain

$$\phi_i{}^I = [(-1)^I / 4(s)^{1/2}] B^{II'} C_{ij} G_j{}^{I'}(t,s,u) , \tag{11}$$

where

$$C_{ij} = \begin{bmatrix} 0 & \dfrac{-4m^2(2t+u)}{t+u} & \dfrac{4m^2u - 2s(4m^2-s)}{t+u} & \dfrac{(2s-4m^2)(t+u) - 4m^2t}{t+u} & \dfrac{4m^2u}{t+u} \\ t & \dfrac{(2s-4m^2)(2u+t)}{t+u} & \dfrac{4m^2(t+2u)}{t+u} & \dfrac{-4m^2t}{t+u} & \dfrac{-st}{t+u} \\ 0 & \dfrac{4m^2u}{t+u} & \dfrac{u(2s-4m^2)}{t+u} & \dfrac{u(2s-4m^2)}{t+u} & \dfrac{4m^2u}{t+u} \\ t & \dfrac{4m^2t}{t+u} & \dfrac{4m^2t}{t+u} & \dfrac{4m^2t}{t+u} & \dfrac{st}{t+u} \\ 0 & \dfrac{-2m(stu)^{1/2}}{t+u} & \dfrac{-2m(stu)^{1/2}}{t+u} & \dfrac{-2m(stu)^{1/2}}{t+u} & \dfrac{-2m(stu)^{1/2}}{t+u} \end{bmatrix} . \tag{12}$$

Equation (11) gives the relation between the helicity amplitudes of the s channel and the G functions of the t channel. Using the relations similar to Eq. (4.33) of Ref. 1 (written for the t channel), we relate the G functions of the t channel to the f functions of the t channel. The result is

$$G_j^{I'}(t,s,u)=D_{jk}f_k^{I'}, \tag{13}$$

where

$$D=\frac{4}{t(s+u)}\begin{pmatrix} s+u & 0 & -4m^2 & u-s & \dfrac{t(u-s)}{4m^2} \\ 0 & 0 & 0 & t & \dfrac{t^2}{4m^2} \\ 0 & 0 & t & 0 & 0 \\ 0 & 0 & 0 & -t & -t \\ 0 & t & 0 & \dfrac{t(s-u)}{s+u} & \dfrac{t(s-u)(t+4m^2)}{4m^2(s+u)} \end{pmatrix}. \tag{14}$$

Combining Eqs. (13) and (14) with (11) and (12), we finally obtain the desired relation between the helicity amplitudes of the s channel and the f functions of the t channel. The result is

$$\phi_i^I=\frac{(-1)^I B^{II'}}{(s)^{1/2}(t+u)(s+u)}K_{ij}f_j^{I'}, \tag{15}$$

where

$$K_{ij}=\begin{pmatrix} 0 & 4m^2u & [4m^2u-2s(t+u)] & \dfrac{-1}{s+u}[4m^2u(s+u)+2s^2t] & \dfrac{4stu}{s+u} \\ (s+u)(t+u) & -st & 4m^2u & \dfrac{4m^2u(s-u)}{s+u} & \dfrac{4stu}{s+u} \\ 0 & 4m^2u & u(s-t-u) & \dfrac{u}{s+u}[4m^2(s+u)+2st] & \dfrac{4stu}{s+u} \\ (s+u)(t+u) & st & -4m^2u & \dfrac{4m^2u(u-s)}{s+u} & \dfrac{-4stu}{s+u} \\ 0 & -2m(stu)^{1/2} & -2m(stu)^{1/2} & \dfrac{2m(stu)^{1/2}(u-s)}{s+u} & \dfrac{(stu)^{1/2}[4m^2u-st]}{m(s+u)} \end{pmatrix}. \tag{16}$$

It is important to realize that a crossing relation of the type of Eq. (15) is completely meaningless, unless one is given a prescription for analytic continuation of the f_j from the t-channel region in which they are defined to the s-channel region in which they are needed. In our particular case this is a trivial matter because the matrix D given in Eq. (14), which relates the f_j to the G functions [see Eq. (13)], is free of branch-point singularities, and therefore the f functions have the same analytic properties as the scalar Mandelstam functions G_i.

We now combine Eqs. (15) and (3) to obtain

$$\sigma_{NN}^I(s)=\frac{4\pi}{p(s)^{1/2}}(-1)^I B^{II'}\,\mathrm{Im}f_2^{I'}(t=0;s). \tag{17}$$

Thus the total s-channel cross section depends only on f_2, one of the spin-triplet $N\bar{N}\to N\bar{N}$ transition amplitudes. Because f_2 is a spin-triplet transition, the quantum numbers characterizing the transition must satisfy[2]

$$CP=PG(-1)^{I'}=+1. \tag{18}$$

Furthermore, the partial-wave amplitudes f_{11}^J contributing to f_2 all have[1]

$$J=L\pm1,$$

so that

$$P(-1)^J=-P(-1)^L=P^2=+1. \tag{19}$$

Equations (18) and (19) show that both C and G are

[2] I. J. Muzinich, Phys. Rev. **130**, 1571 (1963).

redundant in describing the transition and that the "J parity" or signature $(-1)^J$ is equivalent to P. Hence the transition is completely characterized by the four distinct sets of quantum numbers given by $P=\pm1$ and $I'=0$ or 1. We shall therefore define a set of four functions

$$g(P,I';s)=-P\,\mathrm{Im}f_2(P,I';t=0,s)\,,\qquad (P=\pm,\,I'=0\text{ or }1)\,. \quad (20)$$

(The factor $-P$ is merely for the sake of convenience.) Substituting Eq. (20) in Eq. (17), we obtain

$$\sigma_{NN}{}^I(s)=\frac{4\pi}{p(s)^{1/2}}(-1)^{I+1}\sum_{\substack{I'=0,1\\P=\pm1}}B^{II'}Pg(P,I';s)\,, \quad (21)$$

from which we have explicitly

$$\sigma_{pp}(s)=\frac{2\pi}{p(s)^{1/2}}[g(+,0;s)-g(-,0;s)-g(-,1;s)+g(+,1;s)] \quad (22a)$$

and

$$\sigma_{pn}(s)=\frac{2\pi}{p(s)^{1/2}}[g(+,0;s)-g(-,0;s)+g(-,1;s)-g(+,1;s)]. \quad (22b)$$

It is now a simple matter to obtain $\sigma_{\bar{p}p}$ and $\sigma_{\bar{p}n}$ or, equivalently, $\sigma_{p\bar{n}}$. For example, the contribution of a transition determined by $g(P,I';s)$ to $\sigma_{pp}(s)$ is

$$[2\pi/p(s)^{1/2}]Pg(P,I';s)\,.$$

By application of the line-reversal argument of Sharp and Wagner,[3] the contribution of this transition to $\sigma_{\bar{p}p}$ is obtained by using the charge-conjugation operator, so that the contribution to $\sigma_{\bar{p}p}$ is

$$[2\pi/p(s)^{1/2}]g(P,I';s)\,,$$

because here we have $C=P$. We get therefore

$$\sigma_{\bar{p}p}(s)=\frac{2\pi}{p(s)^{1/2}}[g(+,0;s)+g(-,0;s)+g(-,1;s)+g(+,1;s)], \quad (23a)$$

and

$$\sigma_{\bar{p}n}=\sigma_{\bar{n}p}(s)=\frac{2\pi}{p(s)^{1/2}}[g(+,0;s)+g(-,0;s)-g(-,1;s)-g(+,1;s)]. \quad (23b)$$

It should be remembered that Eqs. (22) and (23) are generally valid and are applicable at all energies. They may be particularly useful, however, in the high-energy region in which, as a rule, the theoretical emphasis is on the t-channel amplitudes of definite isospin and parity.

The inversion of Eqs. (22) and (23) provides a set of four fundamental functions directly related to four experimental quantities, namely,

$$\begin{aligned}
g(+,0;s)&=[p(s)^{1/2}/8\pi][\sigma_{pp}+\sigma_{pn}+\sigma_{\bar{p}p}+\sigma_{\bar{p}n}]\,,\\
g(-,0;s)&=[p(s)^{1/2}/8\pi][-\sigma_{pp}-\sigma_{pn}+\sigma_{\bar{p}p}+\sigma_{\bar{p}n}]\,,\\
g(-,1;s)&=[p(s)^{1/2}/8\pi][-\sigma_{pp}+\sigma_{pn}+\sigma_{\bar{p}p}-\sigma_{\bar{p}n}]\,,\\
g(+,1;s)&=[p(s)^{1/2}/8\pi][\sigma_{pp}-\sigma_{pn}+\sigma_{\bar{p}p}-\sigma_{\bar{p}n}]\,.
\end{aligned} \quad (24)$$

Once the energy dependence of the functions $g(P,I;s)$ is known, these functions can be analyzed in terms of any specific model under consideration. The contribution of any model to $g(P,I;s)$ can be obtained directly if it is expressed in terms of the quantum numbers of the t channel, or by an application of the known crossing matrices.

III. AN EXAMPLE: APPLICATION TO THE REGGE MODEL

We illustrate here the use of Eqs. (24) in the Regge model. As we have already mentioned, the f functions satisfy a Mandelstam representation, and therefore, the Froissart and Gribov analytic continuation can be defined on their partial waves. Here we shall, of course, be interested only in f_2. From an equation similar to (4.25b) of Ref. 1 [written for the t channel and absorbing a factor like $(p/2E)$ into $f_{11}{}^J$], we obtain

$$f_2(t;s)=\sum_{J=0}^{\infty}(2J+1)f_{11}{}^J(t)P_J(z)\,, \quad (25)$$

where $z=-1-2s/(t-4m^2)$. Because the signature is determined by parity, here the sum over J runs over even or odd values of J, depending on whether the parity of the state under consideration is even or odd. By application of the Sommerfeld-Watson transformation, one obtains for each Regge pole a contribution of the form

$$f_2(t,s)=\beta(t)(2\alpha+1)P_\alpha(-z)(1+Pe^{-i\pi\alpha})/\sin\pi\alpha\,. \quad (26)$$

We write $\beta(t)=B(t)e^{i\pi\alpha}$, where $B(t)$ is the modified residue and—as usual—$B(t)$ is real below the threshold of the t channel. We have also, to a good approximation,[4]

$$e^{i\pi\alpha}P_\alpha(-z)=P_\alpha(z)\,. \quad (27)$$

Further, at $t=0$, $z=1+T/m$. Thus Eq. (26) becomes

$$f_2(t=0,s)=B(2\alpha+1)P_\alpha(1+T/m)(1+Pe^{-i\pi\alpha})/\sin\pi\alpha\,, \quad (28)$$

where α and B are evaluated at $t=0$. Combining Eq. (28) with Eq. (20) of the previous section, we obtain the contribution of a Regge pole belonging to a given family of trajectories with quantum numbers P and I' to $g(P,I')$, as

$$g(P,I';s)=B_{P,I'}(2\alpha_{P,I'}+1)P_{\alpha_{P,I'}}(1+T/m)\,. \quad (29)$$

[3] D. H. Sharp and W. G. Wagner, Phys. Rev. **128**, 2899 (1962).

[4] *Bateman Manuscript Project*, edited by H. Erdelyi (McGraw-Hill Book Company, Inc., New York, 1953), Vol. I, pp. 140 and 164.

If several Regge poles with the same quantum numbers are considered, and if cuts and background integrals are included, then the $g(P,I;s)$ represent the total contribution of the Regge family with the given P and I. It is only in the high energy region that Eq. (29) might be expected to be adequate with just the highest ranking trajectory included.

The correspondence with the usual trajectory families is

$$g(+,0) \to \text{Pomeranchuk family},$$

$$g(-,0) \to \omega \text{ family},$$

$$g(-,1) \to \rho \text{ family},$$

and

$$g(+,1) \to R \text{ family}.$$

Note that of the twelve possible sets of trajectory quantum numbers for the $N\bar{N}$ system,[2] only the above four contribute to f_2 and therefore to the total s-channel cross sections. The R trajectory[5] has not usually been included in Regge-pole analyses, as there is no known resonance with its quantum number, $I(J^{PG}) = 1(J_{\text{even}}{}^{+-})$. However, in a systematic analysis it should be included, and Eqs. (24) would indicate whether its effect is negligible or not.

[5] A. Pignotti, Phys Rev. **134**, B630 (1964); A. Ahmadzadeh, Phys. Rev. **134**, B633 (1964).

IV. CONCLUSION

The use of Eqs. (24) offers a simple and systematic scheme for analyzing theoretical models in terms of experimental total cross sections. The functions $g(P,I;s)$, constructed from the experimental cross sections, have a fundamental physical significance because they are directly related to one of the $N\bar{N} \to N\bar{N}$ spin-triplet amplitudes in a given state of parity P and isospin I.

It is hoped that the above-mentioned results will act as a spur toward the measurement of the $\bar{p}n$ or, equivalently, $\bar{n}p$ total cross sections.

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Elastic Scattering of Positive Pions by Protons in the Energy Range 500–1600 MeV*

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Differential cross sections for the elastic scattering of positive pi mesons by protons were measured at the Berkeley Bevatron at pion laboratory kinetic energies between 500 and 1600 MeV. Fifty scintillation counters and a matrix coincidence system were used to identify incoming pions and detect the recoil proton and pion companions. Results were fitted with a power series in the cosine of the center-of-mass scattering angle, and total elastic cross sections were obtained by integrating under the fitted curves. The coefficients of the cosine series are displayed, plotted versus the laboratory kinetic energy of the pion. The most striking features of these curves are the large positive value of the coefficient of $\cos^6\theta^*$, and the large negative value of the coefficient of $\cos^4\theta^*$, both of which maximize in the vicinity of the 1350-MeV peak in the total cross section. These results indicate that the most predominant state contributing to the scattering at the 1350-MeV peak has total angular momentum $J=\frac{7}{2}$, since the coefficients for terms above $\cos^6\theta^*$ are negligible at this energy. One possible explanation is that the 1350-MeV peak is the result of an $F_{7/2}$ resonance lying on the same Regge-pole trajectory as the $(\frac{3}{2},\frac{3}{2})$ resonance near 195 MeV.

I. INTRODUCTION

THIS experiment constitutes a portion of an extensive study of the phenomenology of the $\pi - N$ interaction in the energy region above the well-known $(\frac{3}{2},\frac{3}{2})$ resonance occurring at the pion kinetic energy of 195 MeV ($\approx$1236-MeV total c.m. energy for the $\pi - N$ system). The features of outstanding interest are indicated by the cross-section variations displayed in Fig. 1, based on measurements by several experimental groups.[1]

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[1] See for example: H. C. Burrowes, D. O. Caldwell, D. H. Frisch, D. A. Hill, D. M. Ritson, R. A. Schulter, and M. A. Wahlig, Phys. Rev. Letters **2**, 119 (1959); T. J. Devlin, B. J. Moyer, and V. Perez-Mendez, Phys. Rev. **125**, 690 (1962); J. C. Brisson, J. F. Detoeuf, P. Falk-Vairant, L. Van Rossum, and G. Valladas, Nuovo Cimento **19**, 210 (1961); M. J. Longo, J. A. Helland, W. N. Hess, B. J. Moyer, and V. Perez-Mendez, Phys. Rev. Letters **3**, 568 (1959).